Exercise 36

The table gives the number $N(t)$, measured in thousands, of minimally invasive cosmetic surgery procedures performed in the United States for various years t.

t	$N(t)$ (thousands)
2000	5,500
2002	4,897
2004	7,470
2006	9,138
2008	10,897
2010	11,561
2012	13,035

Source: American Society of Plastic Surgeons

(a) What is the meaning of $N'(t)$? What are its units?

(b) Construct a table of estimated values for $N'(t)$.

(c) Graph N and N'.

(d) How would it be possible to get more accurate values for $N'(t)$?

Solution

$N'(t)$ is the rate at which the number of surgeries is increasing with respect to time (units of thousands/year). To obtain the values of $N'(t)$, calculate the slope of the secant line going through two adjacent t values. At $t = 2000$, for example,

$$N'(t) = \frac{N(2002) - N(2000)}{2002 - 2000} = \frac{4{,}897 - 5{,}500}{2} = -301.50.$$

At $t = 2002$, there are two secant lines.

$$N'(t) = \frac{N(2002) - N(2000)}{2002 - 2000} = \frac{4{,}897 - 5{,}500}{2} = -301.50$$

$$N'(t) = \frac{N(2004) - N(2002)}{2004 - 2002} = \frac{7{,}470 - 4{,}897}{2} = 1286.50$$

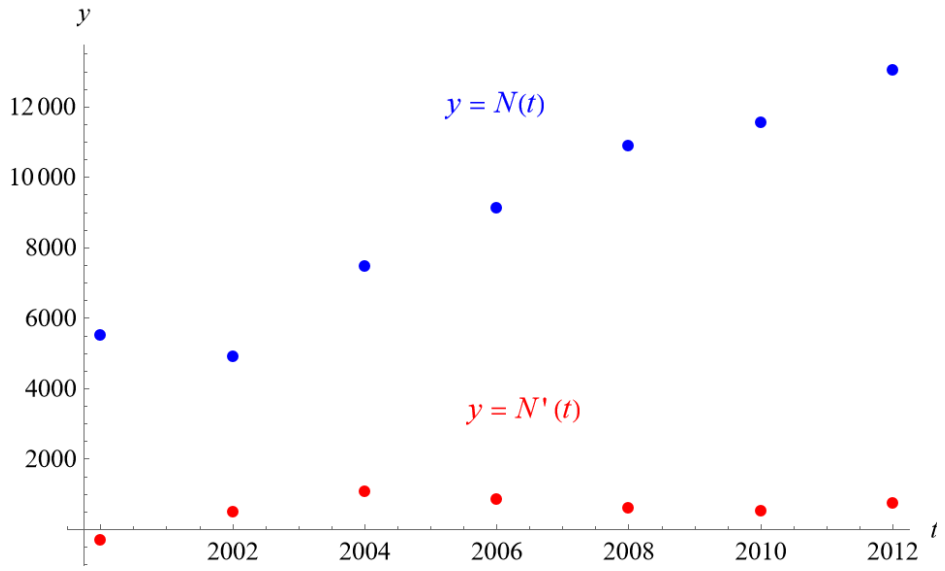
At such times where there are two possible secant lines, take the average for the best estimate.

$$\frac{(-301.50) + (1286.5)}{2} = 492.50$$

Below is a table of estimated values for $N'(t)$.

t	$N(t)$	$N'(t)$
2000	5,500	−301.50
2002	4,897	492.50
2004	7,470	1060.25
2006	9,138	856.75
2008	10,897	605.75
2010	11,561	534.50
2012	13,035	737.00

Below is a graph of N and N' versus t.



To get more accurate values for $N'(t)$, get data from every year rather than every two years.